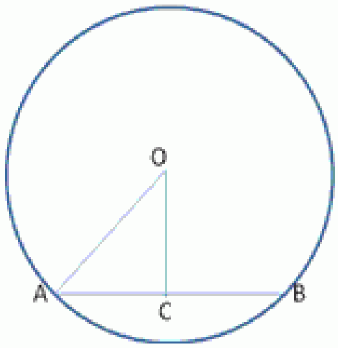
Chapter 17. Circles

Ex 17.1

Answer 1.

(i)



AC = CB ----(1) (Perpendicular from centre to a chord bisects the chord)

In right ΔACO,

By Pythagoras theorem, $OA^2 = OC^2 + AC^2$

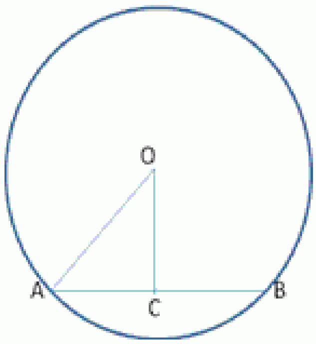
$$13^2 - 12^2 = AC^2$$

$$AC^2 = 169 - 144 = 25$$

$$AC = 5cm$$

$\therefore$ length of chord AB = 2AC (from (1))

(ii)



AC = CB ----(1) (Perpendicular from centre to a chord bisects the chord)

In right ΔACO,

By Pythagoras theorem, $OA^2 = OC^2 + AC^2$

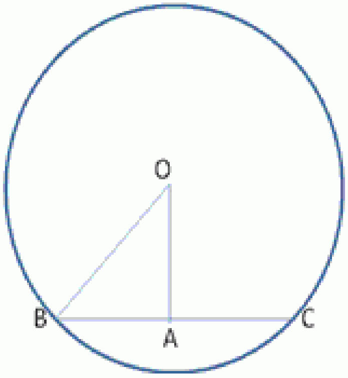
$$AC^2 = (1.7)^2 - (1.5)^2$$
$$= 2.89 - 2.25$$
$$= .64$$

$AC = 0.8cm$

$\therefore$ length of chord $AB = 2AC$ (from (1))

$= 2(0.8) = 1.6cm$

(iii)



$BA = AC$ ----(1) (Perpendicular from centre to a chord bisects the chord)

In right ΔOAB,

By Pythagoras theorem, $OB^2 = OA^2 + AB^2$

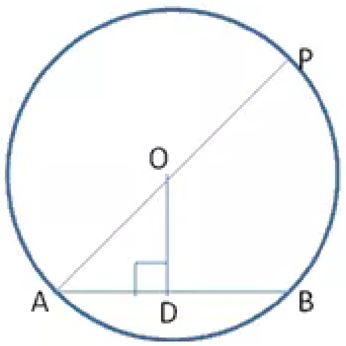
$$AB^2 = 6.5^2 + 2.5^2$$
$$= 42.25 - 6.25$$
$$= 36$$

$AB = 6cm$

$\therefore$ length of chord $BC = 2AB$ (from (1))

$= 2(6) = 12cm$

Answer 2.



AD = DB = 1.6cm (Perpendicular from centre to a chord bisects the chord)

In right Δ ODA,

By Pythagoras theorem, $OA^2 = OD^2 + AD^2$

$$= 1.6^2 + 1.2^2$$

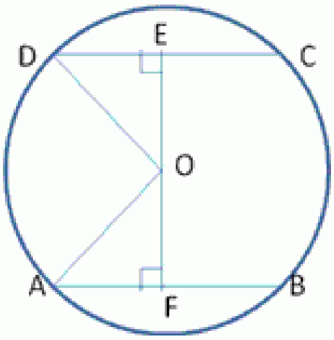
$$= 2.56 + 1.44$$

$$OA^2 = 4$$

OA = 2cm

Diameter(AP) = 2(OA) = 2(1) = 4cm

Answer 3.



AF = FB = 8.4cm

And DE = EC ----(1) (Perpendicular from centre to a chord bisects the chord)

In right ΔODA,

By Pythagoras theorem, $OA^2 = OF^2 + AF^2$

$= (11.2)^2 + (8.4)^2$

$= 125.44 + 70.56$

$OA^2 = 196$

OA = 14cm

OA = OD = 14cm (radii of same circle)

Similarly, In Δ DEO

$OD^2 = OE^2 + DE^2$

$DE^2 = 14^2 + 8.4^2$

$= 196 - 70.56$

$DE^2 = 125.44$

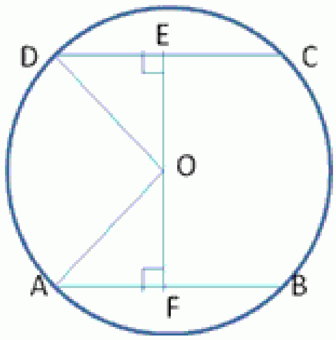
DE = 11.2cm

$\therefore$ length of chord DC = 2DE

= 2(11.2)

= 22.4cm

Answer 4.



AF = FB = 3cm

CE = ED = 7.2cm (Perpendicular from centre to a chord bisects the chord)

In right ΔAFO, By Pythagoras theorem,

$$OA^2 = OF^2 + AF^2$$
$$OA^2 = (7.2)^2 + (3)^2$$
$$OA^2 = 51.84 + 9$$
$$OA^2 = 60.84$$
$$OA = 7.8cm$$

OA = OC = 7.8cm (radii of same circle)

Similarly, In right ΔOFC,

$$OC^2 = OE^2 + EC^2$$
$$OE^2 = (7.8)^2 - (7.2)^2$$

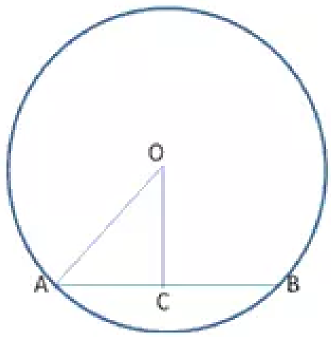
$$= 60.84 - 51.84$$

$$OE^2 = 9$$

$$OE = 3cm$$

Distance from centre of chord CD with length 14.4cm is 3cm.

Answer 5.



AC = CB = 4cm (Perpendicular from centre to a chord bisects the chord)

In rightΔ ABO,

By Pythagoras theorem, $OA^2 = OC^2 + AC^2$

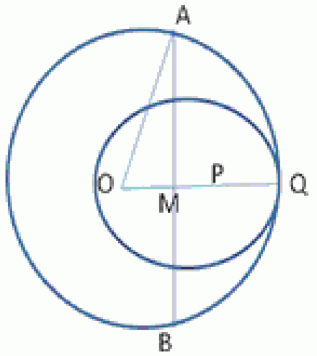
$$OC^2 = 6^2 + 4^2$$

$$OC = 36 - 16 = 20$$

$$OC^2 = 2\sqrt{5}cm$$

Perpendicular distance of chord from centre is $2\sqrt{5}cm$

Answer 6.



OA = OQ = 5cm (Radius of bigger circle)

PQ = 3cm (Radius of smaller circle)

OP = 2cm

Perpendicular bisector of OP, i.e. AB meets OP at M.

$$OM = MP = \frac{1}{2}OP = 1cm$$

In right Δ OMA,

By Pythagoras theorem,

$$OA^2 = OM^2 + MA^2$$

$$MA^2 = 5^2 - 1^2$$

$$= 25 - 1$$

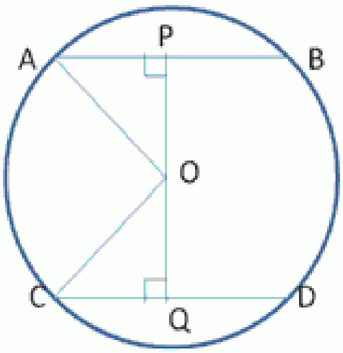
$$= 24$$

$$AM = 2\sqrt{6}cm$$

$$AM = MB = 2\sqrt{6}\ cm$$

$$AB = AM + MB = 2\sqrt{6} + 2\sqrt{6} = 4\sqrt{6}$$

Answer 7.



AP = PB = 3cm

CQ = QD = 6cm (Perpendicular from centre to a chord bisects the chord)

OA = OC = r (say)

Let OP = x, ∴ OQ = 3 - x

In right ΔOQC,

By Pythagoras theorem,

$$OC^2 = OQ^2 + CQ^2$$

$r^2 = (3-x)^2 + 6^2$ ----(1)

Similarly, In ΔOPA,

$OA^2 = AP^2 + PO^2$

$r^2 = 3^2 + x^2$ ----(2)

From (1) and (2)

$$(3-x)^2 + 6^2 = 3^2 + x^2$$

$$-6x + 36 = 0$$

$$x = 6$$

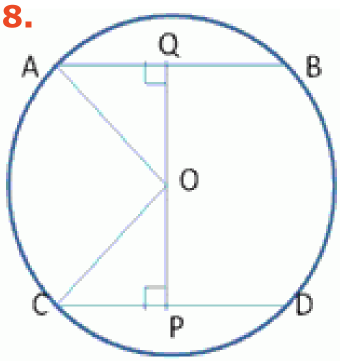
from (2)

$r^2 = 3^2 + 6^2 = 9 + 36 = 45$

$r = 3\sqrt{5}$

Thus, radius of the circle is $3\sqrt{5}$ cm

Answer 8.



CP = PO = 12cm

Let OA = OC = r (say)

Also, let OQ = x, ∴ OP = 17 − x

In right Δ OPC,

By Pythagoras theorem,

$OC^2 = OP^2 + PC^2$

$r^2 = (17 - x)^2 + 12^2$ ----(1)

Similarly, In Δ OQA,

$OA^2 = AQ^2 + QO^2$

$r^2 = 5^2 + x^2$ ----(2)

From (1) and (2)

$(17 - x)^2 + 12^2 = 5^2 + x^2$

289 − 34x + 144 − 25 = 0

34x = 408

x = 12

From (2)

$r^2 = 5^2 + 12^2$

$25 + 144 = 169$

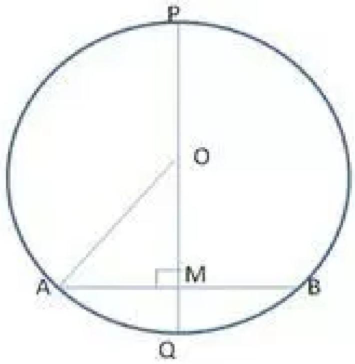
$r = 13$

The radius of the circle is 13cm.

Answer 9.

Given: AB = 18cm, MQ = 3cm

To find: PQ



OQ = OA = r cm(say)

$\therefore$ OM = OQ = MQ = (r − 3)cm

AM = MB = 9cm (PQ $\perp$ AB)

In right ΔOMA,

$$OM^2 + MA^2 = OA^2$$
$$\Rightarrow (r-3)^2 + 9^2 = r^2$$
$$\Rightarrow r^2 - 6r + 9 + 81 = r^2$$
$$\Rightarrow 6r = 90$$
$$\Rightarrow r = 15cm$$

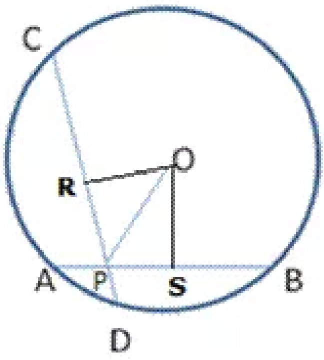
PQ = 2r

(Perpendicular bisector of a chord passes through the centre of the circle)

PQ = 2(15)

PQ = 30cm

Answer 10.



Draw perpendiculars OR and OS to CD and AB respectively.

In triangle ORP and triangle OSP

OP = OP

OR = OS (Distance of equal chords from centre are equal)

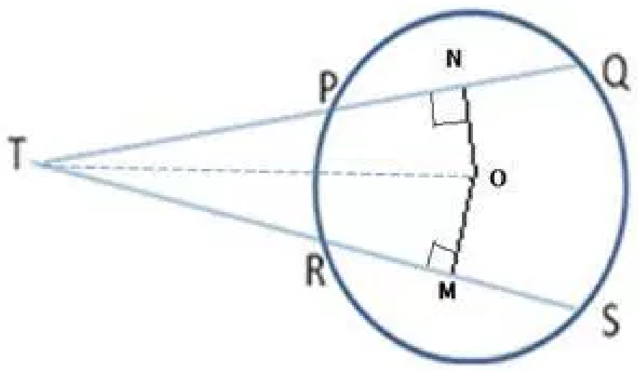
$\angle PRO = \angle PSO$ (right angles)

Therefore, $\Delta ORP \cong \Delta OSP$

Hence, $\angle RPO = \angle SPO$

Thus OP bisects $\angle CPB$.

Answer 11.



Given: PQ = RS

To Prove: TP = TR and TQ = TS.

Construction: Draw $ON \perp PQ$ and $OM \perp RS$.

Proof: Since equal chords are equidistant from the circle therefore

$$PQ = RS \Rightarrow ON = OM \qquad \dots (1)$$

Also perpendicular drawn from the centre bisects the chord.

So, $PN = NQ = \frac{1}{2}PQ$ and $RM = MS = \frac{1}{2}RS$

But $PQ = RS$, we get

$PN = RM$ ...(2)

And, $NQ = MS$ ... (3)

Now in ΔTMO and ΔTNO,

$TO = TO$ (Common)

$MO = NO$ (By (1))

$\angle TMO = \angle TNO$ (Each 90 degrees)

Therefore, $\Delta TMO \cong \Delta TNO$ (By RHS)

$\Rightarrow TN = TM$ (By CPCT) .... (4)

Subtracting, (2) from (4), we get

$TN - PN = TM - RM$

$\Rightarrow TP = TR$

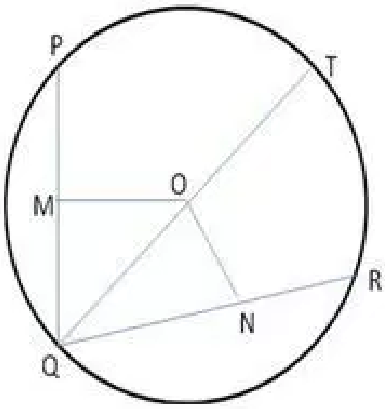
Adding (3) and (4), we get

$TN + NQ = TM + MS$

$\Rightarrow TQ = TS$

Hence Proved.

Answer 12.



Let QT be the diameter of $\angle PQR$

Since, $PQ = QR$

$\therefore OM = ON$

In ΔOMQ and ΔONQ

$OM = ON$ (equal chords are equidistant from the centre)

$\angle OMQ = \angle ONQ$ (90° each)

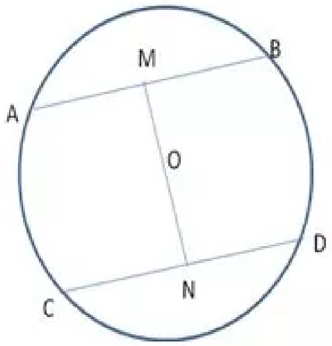
$OQ = OQ$ (common)

$\Delta OMQ \cong \Delta ONQ$ (RHS)

$\therefore \angle OQM = \angle OQN$ (CPCT)

Thus QT i.e. diameter of the circle bisects $\angle PQR$

Answer 13.



AM = MB

CN = ND

$\therefore OM \perp AB$

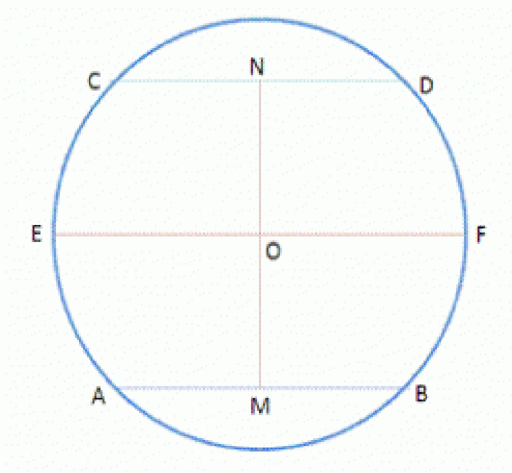
and $ON \perp CD$ (A line bisecting the chord and passing through the centre of the circle is perpendicular to the chord)

$\therefore \angle OMA = \angle OND = 90^\circ$ each

But these are alternate interior angles

$\therefore AB \parallel CD$

Answer 14.



Given: AB and CD are two chords of a circle with centre O.

AB||CD, M and N are midpoints of AB and CD respectively.

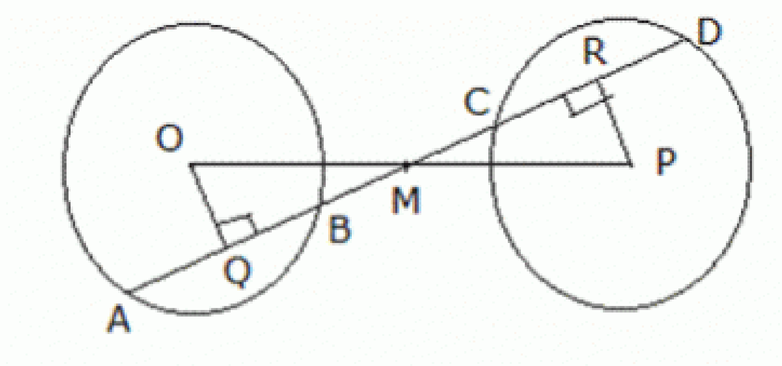
To prove: MN passes through centre O.

Construction: Join OM, ON, and through O, draw a straight line EF parallel to AB.

Proof: OM^ AB (line joining the midpoint of a chord to the centre of a circle is perpendicular to it)

\ ÐAMO = 90°
Q ÐMOE = 90° [cointerior angle of ÐAMO]
Þ ÐNOE = 90° [corresponding angle of ÐAMO]
\ ÐMOE + ÐNOE = 180°
Þ MON is a straight line.
Hence MN passes through centre O.

Answer 15.



Given: Two congruent circles with centre O and P. M is the mid-point of OP

To prove: Chord AB and CD are equal.

Construction: Draw $OQ \perp AB$ and $PR \perp CD$.

Proof: In ΔOQM and ΔPRM

$\angle OQM = \angle PRC$ (Each 90°)

$OM = MP$ (As M is the mid-point)

$\angle OMQ = \angle PMR$ (Vertically opposite angles)

Therefore, $\Delta OQM \cong \Delta PRM$ (By AAS)

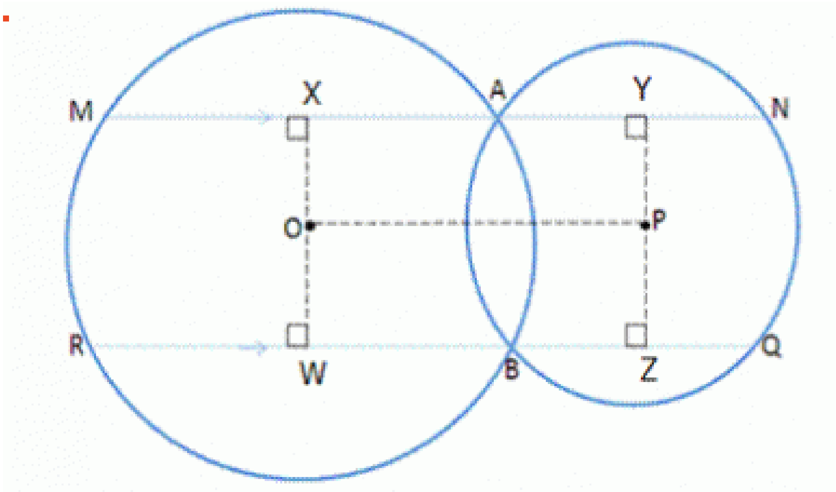
$\Rightarrow OQ = PR$ (By CPCT)

Now the perpendicular distances of two chords in two congruent circles are equal, therefore chords are also equal.

$\Rightarrow AB = CD$.

Hence Proved.

Answer 16.



Given: Two circles with centres O and P, and MN||OP||RQ

To prove: (i) MN = 2OP (ii) MN = RQ.

Construction: OX ⊥ MN, PY ⊥ MN, OW ⊥ RZ, PZ ⊥ RQ

Proof: Since each angle of the quadrilateral XYZW is a right angle, XYZW is a rectangle.

Also, XYPO is a rectangle. ...(1)

Now, perpendicular drawn from the centre to the chord bisects the chor

Therefore, MA = 2 XA and AN = 2 AY ...(2)

Now, MN = MA + AN = 2 XA + 2 AY [from (2)]

⇒ MN = 2(XA + AY) = 2 XY

⇒ MN = 2 OP [As XYPO is a rectangle, XY = OP] ... (3)

This proves part (i).

By similar arguments, we have RQ = 2 OP ... (4)

Using (3) and (4), we get

MN = RQ.

This proves part (ii).

Answer 17.

ABC is an equilateral triangle,

$\therefore\ AB = AC$

Also $AN = MB$ (radii of same circle)

$\Rightarrow NC = MB$

In ΔBNC and ΔCMB

$NC = MB$ (proved above)

$\angle B = \angle C$ (60°each)

$BC = BC$ (Common)

$\therefore\ \Delta BNC \cong \Delta CMB$ (SAS)

$\therefore\ BN = CM$ (CPCT)

Answer 18.

In ΔDAM and ΔBAN

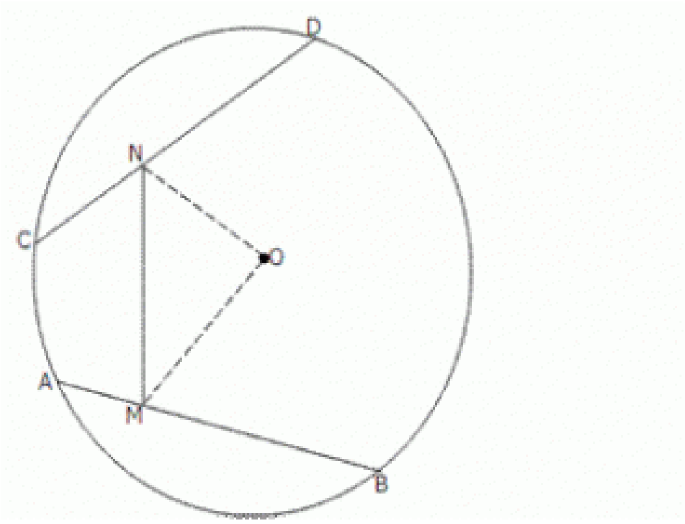
$AN = AM$ (radii of same circle)

$AD = AB$ (sides of square ABCD)

$\angle DAM = \angle BAN$ (Common)

$\therefore\ \Delta DAM \cong \Delta BAN$ (SAS)

Answer 19.



and N are mid points of equal chords AB and CD respectively.

N $\perp$ CD and OM $\perp$ AB

$\angle ONC = \angle OMA$ (90° each) ----(1) (A line bisecting the chord and passing rough the centre of the circle is perpendicular to the chord)

AB = CD

ON = OM (equal chords are equidistant from the centre)

ΔMON,

MO = NO

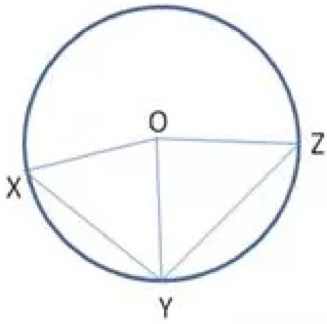
$\therefore \angle ONM = \angle OMN$ ----(2)

ubtracting (2) from (1)

$ONC - \angle ONM = \angle OMA - \angle OMN$

$\angle CNM = \angle AMN$

Answer 20.



Join OX and OZ

In ΔXOY and ΔZOY

OX = YZ (radii of same circle)

XY = YZ (given)

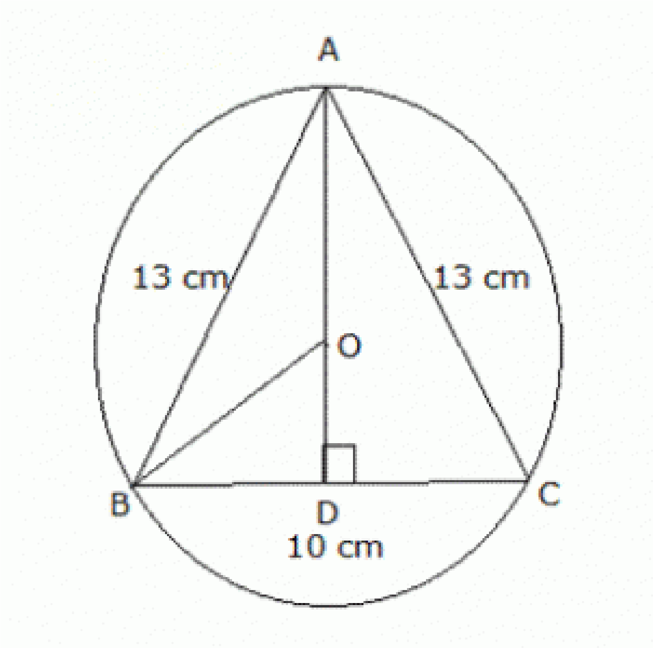
OY = OY (common)

$\therefore$ $\Delta XOY \cong \Delta ZOY$ (SSS)

$\therefore$ $\angle OYX = \angle OYZ$ (CPCT)

Hence, OY is the bisector of $\angle XYZ$ passing through O

Answer 21.



ince ABC is an isosceles triangle, AOD is the perpendicular bisector of BC.

n triangle ADC, by Pythagoras theorem we have

$D^2 = AC^2 - DC^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow AD = 12$ cm $\Rightarrow AO + OD = 12 \Rightarrow AO = 12 - x$ (Assuming $OD = x$ cm)

gain in triangle OBD,

$O^2 = BD^2 + OD^2 = 25 + x^2$ (As BD = 5 cm)

$\Rightarrow (12 - x)^2 = 25 + x^2$ (As AO = BO = radius)

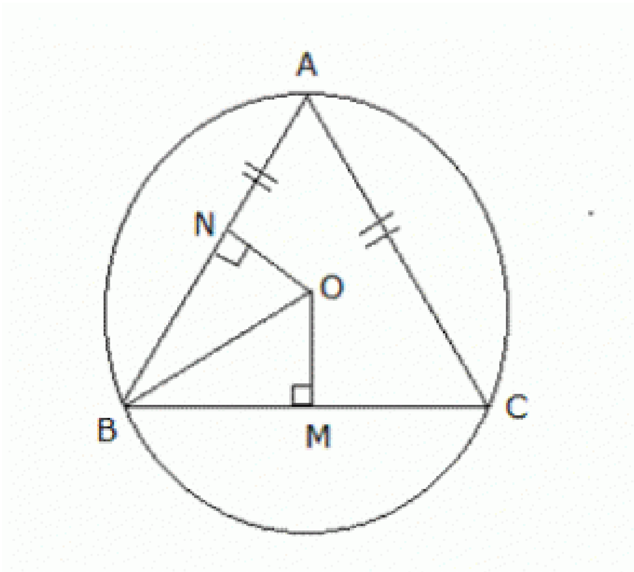
$\Rightarrow 144 + x^2 - 24x = 25 + x^2$

$\Rightarrow -24x = 25 - 144 = -119$

$\Rightarrow x = 4.96$ cm

$\Rightarrow AO = 12 - 4.96 = 7.04$ cm

Answer 22.



Given: AB = AC, $\angle ABO = \angle CBO$

To Prove: AB = BC

Construction: Draw ON $\perp$ AB and OM $\perp$ BC

Proof: In triangles BNO and BMO,

$\angle NBO = \angle MBO$ (Given)

$\angle BNO = \angle BMO$ (Each 90°)

BO = BO (Common)

Thus, $\Delta BNO \cong \Delta BMO$ (By AAS)

$\Rightarrow BN = BM$

$\Rightarrow 2BN = 2BM$ (Since perpendicular drawn from the centre bisects the chord)

$\Rightarrow AB = BC$

Hence Proved.

Ex 17.2

Answer 1.

Since arc AB makes $\angle AOB$ at the centre and $\angle APB = 50^\circ$ on the remaining part of the circle.

$\angle AOB = 2\angle APB$

$\angle AOB = 2(50)$

$= 100^\circ$

$AO = OB = x$ (radii of same circle)

In ΔAOB

$\angle AOB + \angle BAO + \angle ABO = 180$

$180 + x + x = 180$

$2x = 80$

$x = 40$

$\therefore \angle OAB = 40^\circ$

Answer 2.

$\angle AOC = 150^\circ$

Reflex $\angle AOC = 360^\circ - 150^\circ = 210^\circ$

$\angle ABC = \frac{1}{2}$ reflex $\angle AOC = \frac{1}{2}(210^\circ)$

$\angle ABC = 105^\circ$

Answer 3.

BOC is the diameter of circle,

$\therefore \angle BOC = 180°$

Since arc BC makes $\angle BOA$ at the centre and $\angle BAC$ on the remaining part of the circle

$\therefore \angle BAC = \frac{1}{2}\angle BOC$

$\therefore \angle BAC = \frac{1}{2}(180)$

$= 90°$

Answer 4.

Since arc BC makes $\angle BOC$ at the centre and $\angle BDC$ on the remaining part of the circle

$\therefore \angle BDC = \frac{1}{2}\angle BOC = \frac{1}{2}(x) = \frac{1}{2}x$

$\angle BDC = \angle BEC = \angle\frac{x}{2}$ (angles in the same segment)

$\angle ADB = AEP = 180 - \angle\frac{x}{2}$

Also, $\angle BPC = \angle DPE = \angle y$ (vertically opposite)

In quadrilateral ADPE,

$\angle ADP + \angle DEP + \angle PEA + \angle EAD = 360°$

$180 - \angle\frac{x}{2} + \angle y + 180 - \angle\frac{x}{2} + z = 360°$

$-\angle x + \angle y + \angle z = 0$

$\angle x = \angle y + \angle z$

Answer 5.

: Let O be the centre of the circle on diameter AC of the circle

Since, EC make $\angle EOC$ at the centre and $\angle EBC$ on the remaining part of the circle

$\therefore \angle EOC = 2\angle EBC$

$= 2(65)$

$= 130°$

In ΔEOC,

$\angle EOC + \angle OCE + \angle CEO = 180°$

$130 + x + x = 180°$ ($OE = OC$, $\therefore \angle OEC = \angle OCE = x$)

$2x = 50$

$x = 25$

$\angle OCE = \angle OEC = 25°$

Also, $\angle OCE = \angle CED = 25°$ (alternate interior angles)

Answer 6.

$\angle AOB = q$

Reflex $\angle AOB = 360 - q$

Since arc AB subtends reflex $\angle AOB = (360 - q)°$ at the centre and $\angle ACB$ on the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2}(\text{reflex } \angle AOB)$

If OACB is a parallelogram

$\angle AOB = \angle ACB$

$q = p$

$360 - 2p = p$

$3p = 360$

$P = 120°$

Answer 7.

In ΔPQR,

$PQ = PR$

$\therefore \angle PQR = \angle PRQ = 35^\circ$

Also, $\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$35 + 35 + \angle QPR = 180$

$\angle QPR = 110^\circ$

In cyclic quadrilateral PQSR,

$\angle QPR + \angle QSR = 180$

$110 + \angle QSR = 180$

$\angle QSR = 70$

Also, $\angle QSR = \angle QTR = 70^\circ$ (Angles in the same segment)

Answer 8.

In cyclic quadrilateral ABCD,

$\angle BCD + \angle DAB = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$100 + \angle DAB = 180$

$\angle DAB = 80^\circ$

In ΔDAB,

$\angle DAB + \angle ABD + \angle BDA = 180^\circ$

$80 + 70^\circ + \angle BDA = 180^\circ$

$\angle BDA = 30^\circ$

Answer 9.

It is given that $\angle AOC = 100°$

Arc AC subtends $\angle AOC$ at the centre of circle and $\angle APC$ on the circumference of the circle.

$\therefore \angle AOC = 2\angle APC$

$$\Rightarrow \angle APC = \frac{100°}{2} = 50°$$

It can be seen that APCB is a cyclic quadrilateral.

$\therefore\ \angle APC + \angle ABC = 180°$ (Sum of opposite angles of a cyclic quadrilateral)

$\Rightarrow \angle ABC = 180° - 50° = 130°$

Now, $\angle ABC + \angle CBD = 180°$ (Linear pair angles)

$\Rightarrow \angle CBD = 180° - 130° = 50°$

Answer 10.

PQ is a diameter of the circle

$\therefore$ CPRQ $= 90°$ (angle is a semi circle)

$\angle RPQ = 40°$ (given)

In ΔPQR,

$\angle PRQ + \angle RQP + \angle QPR = 180$ (Angle sum property)

$90 + \angle RQP + 40 = 180$

$\angle RQP = 50°$

Answer 11.

$\angle B = 65°$(given)

$\angle B + \angle D = 180$ (Opposite angles of a cydic quadrilateral)

$65 + \angle D = 180$

$\angle D = 115$

Also, AB || CD

$\therefore \angle B + \angle C = 180$ (Sum of angles on same side of transversal)

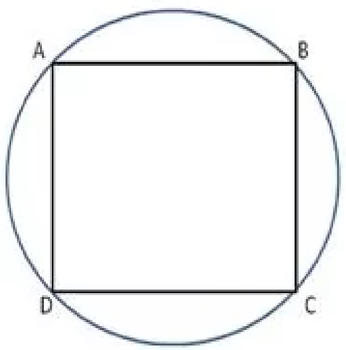
$\angle C = 180 - 65 = 115$

Again, $\angle A + \angle C = 180^{\circ}$ (Opposite angles of a cyclic quadrilateral)

$\angle A = 180 - 115$

$= 65°$

Answer 12.



$m\ \angle A = 3\ (m\ \angle C)$

$\angle A + \angle C = 180$ (Opposite angles of a cyclic quadrilateral)

$3\angle C + \angle C = 180$

$4\ \angle C = 180$

$\angle C = 45$

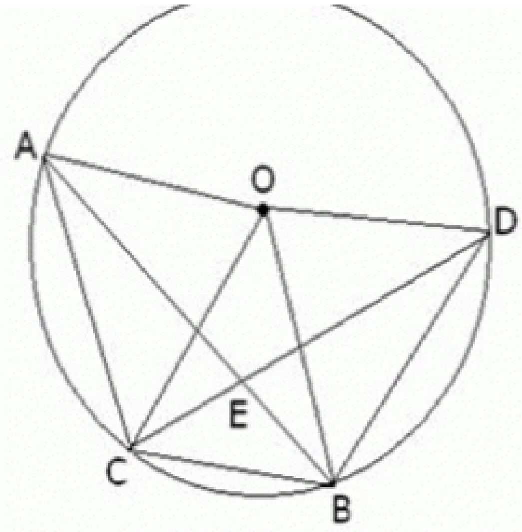
$m\ \angle A = 3(m\ \angle C)$

$= 3 \times 45$

$= 135$

$m\angle A = 135°$

Answer 13.



Arc AC subtends $\angle AOC$ at the centre of circle and $\angle ABC$ on the circumference of the circle.

$\therefore \angle AOC = 2\angle ABC$ ... (1)

Similarly, $\angle BOD$ and $\angle DCB$ are the angles subtended by the arc DB at the centre and on the circumference of the circle respectively.

$\therefore\ \angle BOD = 2\angle DCB$... (2)

Adding (1) and (2),

$\angle AOC + \angle BOD = 2(\angle ABC + \angle DCB)$ ... (3)

In triangle ECB,

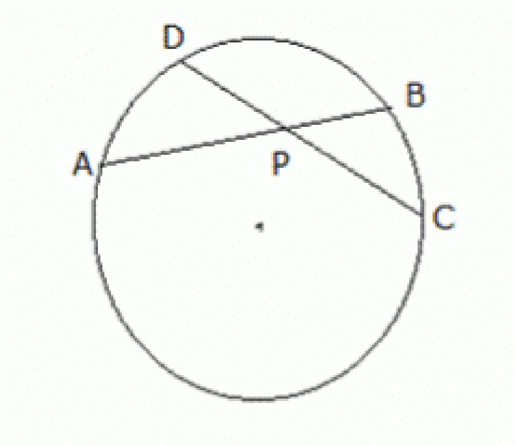
$\angle AEC = \angle ECB + \angle EBC = \angle DCB + \angle ABC$

From (3),

$\angle AOC + \angle BOD = 2\angle AEC$

Hence Proved.

Answer 14.



If two chords of a circle interest internally then the products of the lengths of segments are equal, then

$AP \times BP = CP \times DP$ ...(1)

But, $AP = CP$ (Given) ... (2)

Then from (1) and (2), we have

$BP = DP$ ... (3)

Adding (2) and (3),

$AP + BP = CP + DP$

$\Rightarrow AB = CD$

Hence Proved.

Answer 15.

$\angle NYB = 50°$

$\angle YNB = 20°$

In ΔNYB,

$20° + \angle NBY + 50° = 180°$

$\Rightarrow \angle NBY = 180° - 70° = 110°$

Now, $\angle MAN = \angle NBM = 110°$ (Angles in the same segment)

$\angle MON = 2\angle MAN$ (Arc MN subtends $\angle MON$ at centre and $\angle MAN$ at remaining part of the circle)

$\angle MON = 2(110°) = 220°$

Reflex $\angle MON = 360° - \angle MON$

$= 360° - 220°$

Reflex $\angle MON = 140°$.

Answer 16.

Given AP and AQ are diameters of circles with centre O and O^1 respectively.

$\therefore \angle APB = 90°$ ---(1) (Angle in a semicircle is a right angle)

Similarly, $\angle ABQ = 90°$ ---(2)

Adding (1) and (2)

$\angle APB + \angle ABQ = 90° + 90°$

$\angle PBQ = 180°$

Hence, PBQ is a straight line

$\therefore$ P, B and Q are collinear.

Answer 17.

AB and AC are diameters of circles with centre O and O^1 respectively.

$\therefore \angle ADB = 90^\circ$ ---(1) (Angle in a semi circle is a right angle)

Similarly, $\angle ADB = 90^\circ$ ---(2)

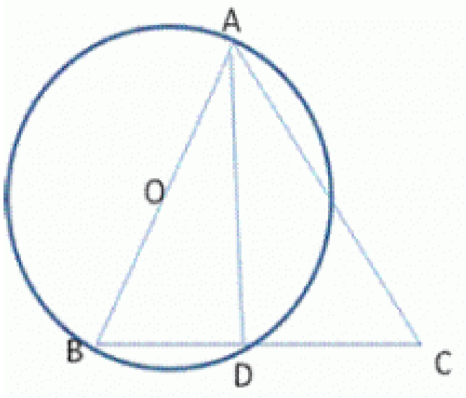
Adding (1) and (2)

$\angle ADB + \angle ADC = 90 + 90$

$\angle BDC = 180^\circ$

Hence, BDC is a straight line.

Answer 18.



BD = DC

AB be the diameter of the circle with centre O.

= 90° (Angles in a semicircle is a right triangle)

$\angle ADC$ = 180 (linear pair)

= $180 - 90 = 90^\circ$

ınd ΔADC

given)

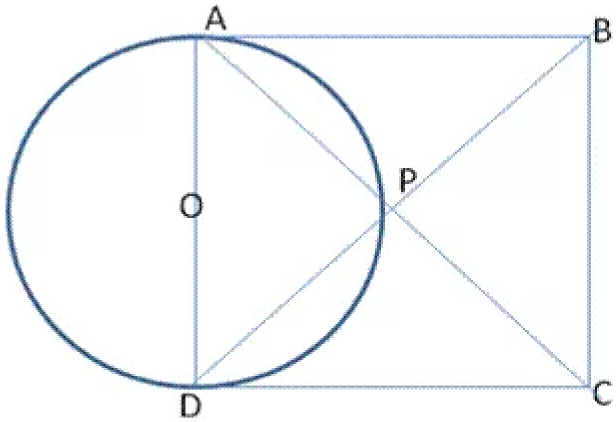
$\angle ADC$ (90°each)

Common)

ΔADC (RHS)

= DC (CPCT)

Answer 19.



We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle APD = 90^\circ$ - (1)

Also, AD is the diameter of the circle with centre O.

$\therefore \angle APD = 90^\circ$ - (2) (Angle in semi circle)

From (1) and (2), we get, The circle drawn with any side of a rhombus as a diameter, passes through point of intersection of its diagonals.

Answer 20.

In cyclic quadrilateral ABCD,

$\angle BAD + \angle BCD = 180^\circ$ - (1)

Opposite angles of cyclic quadrilateral

Also, $\angle BCD + \angle BCE = 180^\circ$ - (2) (Linear pair)

From (1) and (2), we get

$\angle BAD = \angle BCE$

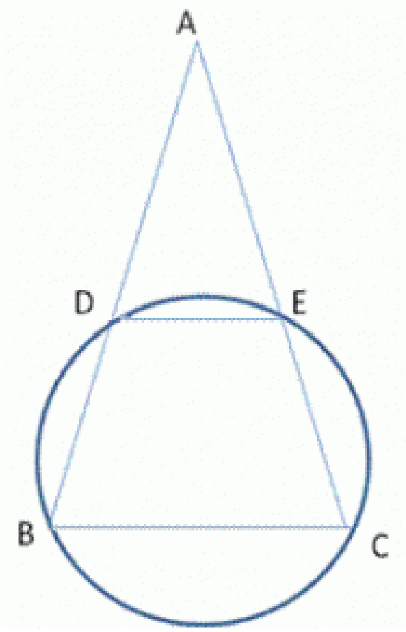
In Δ EBC and Δ EDA

$\angle BAD = \angle BCE$ (proved above)

$\angle BEC = \angle DEA$ (common)

$\therefore \Delta EBC \sim \Delta EDA$ (AA corollary)

Answer 21.



prove = DE ||BC

of: In cyclic quadrilateral DECB

EC + ∠DBC = 80° - (1) (Opposite angles of cyclic quadrilateral)

o, ∠AED + ∠DEC = 80° - (2) (Linear pair)

m (1) and (2), we get,

3C = ∠AED - (3)

= AC (given)

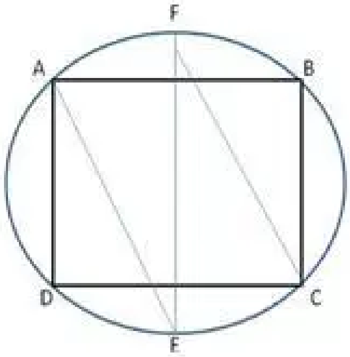
∠ABC = ∠ACB - (4) (angles opposite to equal sides of triangle)

m (3) and (4) ⇒ ∠AED = ∠ACB

; these are corresponding angles.

E || BC

Answer 22.



In cyclic quadrilateral ABCD

$\angle A + \angle C = 180^\circ$

$1/2\angle A + 1/2\angle C = 90^\circ$

$\angle EAB + \angle BCF = 90^\circ$ - (1) (AE bisects $\angle A$; CF bisects $\angle C$)

Also,

$\angle BCF = \angle BAF$ - (2) (Angles in the same segment)

Using (1) in (2) we get,

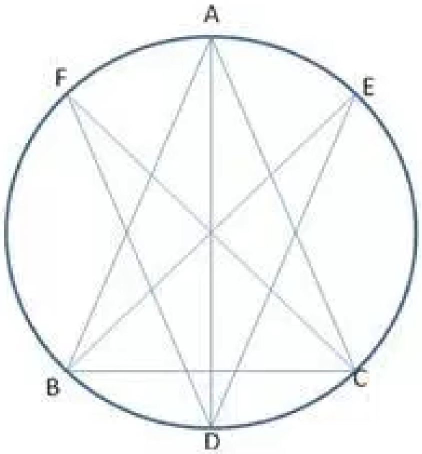
$\angle EAB + \angle BAF = 90^\circ$

$\angle FAE = 90^\circ$

EF is the diameter of the circle,

$\therefore$ angle in a semi circle is a right angle

Answer 24.



Since AD, BE and CF are bisectors of $\angle A$, $\angle B$ and $\angle C$ respectively.

$\therefore \angle 1 = \angle 2 = \angle \frac{A}{2}$

$\angle 3 = \angle 4 = \angle \frac{B}{2}$

$\angle 5 = \angle 6 = \angle \frac{C}{2}$

$\angle ADE = \angle 3$ ----(1)

Also $\angle ADF = \angle 6$ ----(2) (angles in the same segment)

Adding (1) and (2)

$\angle ADE + \angle ADF = \angle 3 + \angle 6$

$\angle D = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

$\angle D = \frac{1}{2}(B + \angle C) = \frac{1}{2}(180 - \angle A)$ $(\angle A + \angle B + \angle C = 180°)$

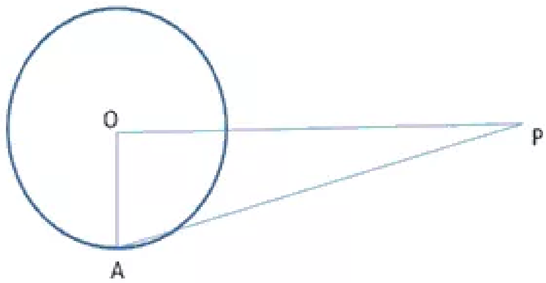
$\angle D = 90 - \frac{1}{2}\angle A$

Similarly,

$\angle E = 90 - \frac{1}{2}\angle B$, $\angle F = 90 - \frac{1}{2}\angle C$

Ex 17.3

Answer 1.



OA ⊥ AP (radius is perpendicular to tangent at the point of contact)

In right Δ OAP,

$OP^2 = OA^2 + AP^2$
$AP^2 = 5^2 - 3^2$

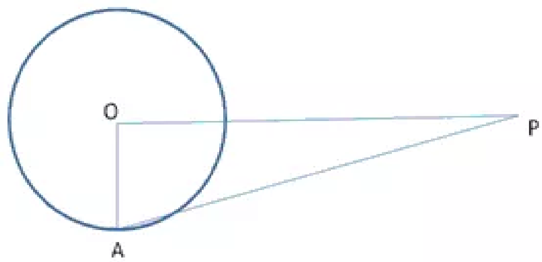
$= 25 - 9$

$= 16$

AP = 4cm

The length of the tangent is 4cm.

Answer 2.



OA ⊥ AP (radius is perpendicular to tangent at the point of contact)

In right ΔOAP,

$OP^2 = OA^2 + AP^2$
$AP^2 = 17^2 + 15^2$

$= 289 - 225$

$= 64$

AP = 8

The radius of the circle is 8cm.

Answer 3.

XP = XQ

AR = AP {Length of tangents drawn from an external point to a circle are

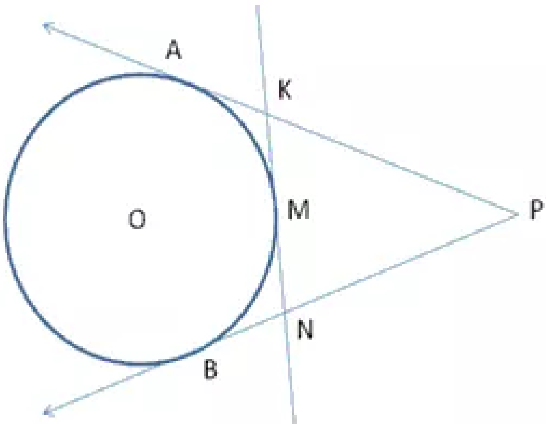
BR = BQ equal}

XP = XQ

XA + AP = XB + BR

XA + AR = XB + BR {Using (1)}

Answer 4.



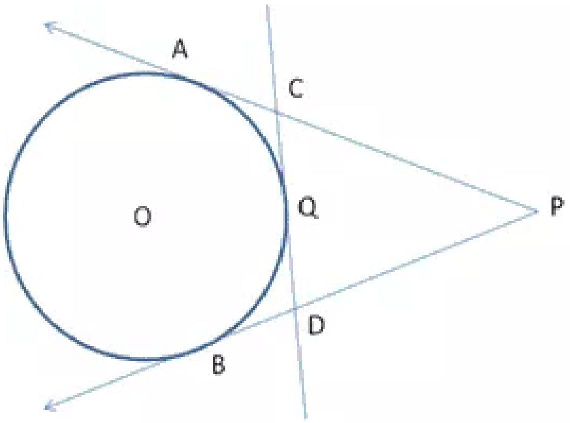
KA = KM ---(1) {Length of tangents drawn from an external point to a

NM = NB circle are equal}

KN = KM + MN

KN = KA + BM {Using (1)}

Answer 5.



PA = PB = 20 Units ---(1) {Length of tangents drawn from an external point

CQ = CA and DQ = DB to a circle are equal}

Perimeter of ΔPCD

= PC + CD + PD

= PC + CQ + QD + PD {Using (1)}

= PA + PB

= 2PA

= 2(20)

= 40 Units

Answer 6.

To prove:- AF + BD + CE = AE + BF + CD

Proof:- AF = AE ----(1) {Length of tangents drawn from an external point

BD = BF ----(2) to a circle are equal }

CE = CD ----(3)

Adding (1), (2) and (3)

AF + BD + CE = AE + BF + CD

Answer 7.

To prove:- $AQ = \frac{1}{2}$ (Perimeter of Δ ABC)

Proof:- $BQ = BR = 5 - r$ ---(1)
$PC = CR = 12 - r$ ---(2)

(1) (Lengths of tangents drawn from an external point to a circle are equal)

Perimeter of Δ ABC = AB + BC + AC

= AB + BP + PC + AC

= AB + BQ + CR + AC Using (1)

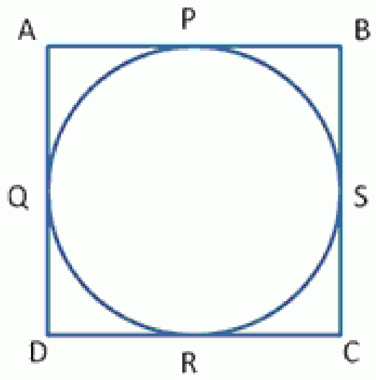
= AQ + AR

= 2 AQ

2 AQ = Perimeter of Δ ABC

$AQ = \frac{1}{2}$ (Perimeter of Δ ABC)

Answer 8.



Let the sides of parallelogram ABCD touch the circle at points P, Q, R and S.

AP = AS - (1)

PB = BQ - (2) (Length of tangents drawn from an external point to a circle a equal)

DR = DS - (3)

RC = CQ - (4)

Adding (1), (2), (3) and (4)

AP + PB + DR + RC = AS + BQ + DS + CQ

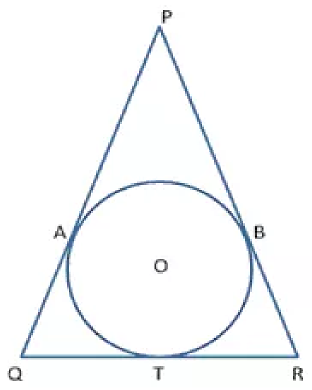
AB + CD = AD + BC

2 AB = 2 BC $\Rightarrow$ AB = BC (Opposite sides of a parallelogram are equal)

$\therefore$ AB = BC = CD = DA,

Hence , ABCD is a rhombus.

Answer 9.



To proof:- QT = TR

Proof: Let the circle touches sides PQ and PR at points A and B respectively.

$$\left.\begin{aligned} PA &= PB \\ AQ &= QT \\ BR &= TR \end{aligned}\right] \text{(Lengths of tangents drawn from an external point to a circle are equal)}$$

Given, PQ = PR

PA + AQ = PB + BR

AQ = BR (Using (1))

$\Rightarrow$ QT = TR

Answer 10.

In ΔAOP = ΔBOP

AP = PB (lengths of tangents drawn from and external point to a circle are equal)

OP = PO (common)

$\angle$PAO = $\angle$PBO = 90° (radius is $\perp$ to tangent at the point of contact)

$\therefore \Delta$ AOP $\cong$ ΔBOP (By RHS)

Δ AOP = ΔBOP (By CPCT)

In ΔAMO and ΔBMO

AO = OB (radius of same circle)

$\angle$MOA = $\angle$MOB (Proved above)

OM = MO (Common)

$\therefore$ ΔAMO $\cong$ Δ BMO (By CPCT)

$\angle$AMO = $\angle$BMO

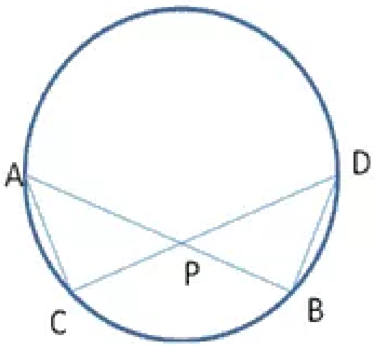
$\angle$AMO + $\angle$BMO = 180°

$\therefore$ 2 $\angle$AMO = 180°

$\angle$BMO = $\angle$AMO = 90°

Hence, OP is the perpendicular bisector of AB.

Answer 11.



Let DP = x cm

In ΔAPC and ΔDPB

$\angle PAC = \angle PDB$ (angles in the some segment)

$\angle APC = \angle DPB$ (vertically opposite angle)

$\therefore \Delta APC \sim \Delta DPB$ (AA corollary)

$\frac{AP}{DP} = \frac{PC}{PB}$ (similar sides of similar triangles)

$\frac{5}{x} = \frac{2.5}{3}$

$\Rightarrow x = \frac{15}{2.5} = \frac{150}{25} = 6cm$

Answer 12.

Let TQ = x cm

In ΔPTR and ΔSTQ

$\angle TPR = \angle TSQ$ (angles in the same segment)

$\angle PTR = \angle STQ$ (vertically opposite $\angle$'s)

$\therefore \angle PTR = \angle STQ$ (AA corollary)

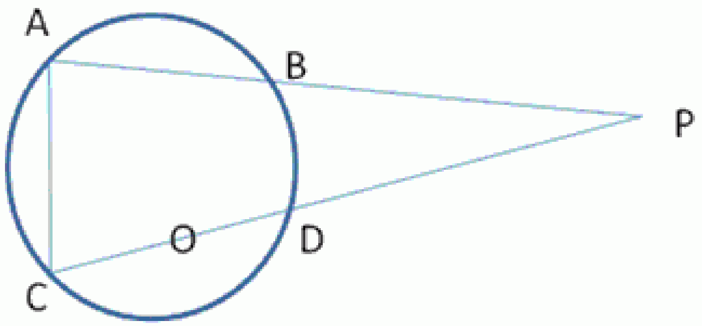
$\frac{PT}{ST} = \frac{TR}{TQ}$ (similar sides of similar triangles)

$\frac{18}{6} = \frac{12}{x}$

$= x = 4$

$\Rightarrow TQ = 4$ cm

Answer 13.



Let OD = OC = r (say)

PO = 14.5 , CP = r + 14.5

PD = 14.5 − r

In ΔBPD and ΔAPC

$\angle BPD = \angle APC$ (Common)

$\angle ABD + \angle DBP = 180^\circ$ ---(1) (Linear pair)

Also, $\angle ABD + \angle ACD = 180^\circ$ ---(2) (Opposite angles of a cyclic quadrilateral)

From (1) and (2)

$\angle DBP = \angle ACD$

$\therefore \Delta BPD \sim \Delta CPA$ (AA corollary)

$$\frac{8}{r + 14.5} = \frac{4.5 - r}{15}$$

$$120^\circ = 14.5^2 - r^2$$

$$r^2 = 210.25 - 120$$

$$r^2 = 90.25$$

$$r = 9.50$$

Radius of the circle is 9.5cm.

Answer 14.

Let PT = x cm

Since, PAB is a secant and PT is a tangent to the given circle, we have,

$PA \cdot PB = PT^2$

$\Rightarrow \; 4 \cdot 9 = PT^2$

$\Rightarrow \; PT^2 = 36$

$\Rightarrow \; PT = 6cm$

Answer 15.

Let PT = x cm

Since, PAB is a secant and PT is a tangent to the given circle, we have,

$PA \cdot PB = PT^2$

$\Rightarrow \; 4 \cdot 9 = PT^2$

$\Rightarrow \; PT^2 = 36$

$\Rightarrow \; PT = 6cm$

Answer 16.

Let PT = x cm

Since, PAB is a secant and PT is a tangent to the given circle, we have,

$PA \cdot PB = PT^2$

$\Rightarrow 4 \cdot 9 = PT^2$

$\Rightarrow PT^2 = 36$

$\Rightarrow PT = 6cm$

Answer 17.

Let OD = OC = x cm (radius of same circle)

Since, PCD is a secant and PT is a tangent to the given circle, we have,

$$PC \cdot PD = PT^2$$

$$3 \cdot (3 + 2x) = 6^2$$

$$\Rightarrow \quad 9 + 6x = 36$$

$$\Rightarrow \quad 6x = 27$$

$$\Rightarrow \quad x = \frac{27}{6} = \frac{9}{2}$$

Radius of the circle is $\frac{9}{2}$cm, diameter is 9cm

Answer 18.

$R_1 = 4cm, R_2 = 12cm$

PQ = 15cm

$AB^2 = PQ^2 + (R_2 - R_1)^2$

$\Rightarrow AB^2 = 15^2 + (12 - 4)^2$

$\Rightarrow AB^2 = 225 + 64$

$\Rightarrow AB^2 = 289$

$\Rightarrow AB = 17cm$

The diameter between the centre is 17cm

Answer 19.

To find: PQ

$R_1 = 3cm, R_2 = 8cm$

$AB = 13cm$

$PQ^2 = AB^2 - (R_2 - R_1)^2$

$\Rightarrow PQ^2 = 13^2 - (8 - 3)^2$

$\Rightarrow PQ^2 = 169 - 25$

$\Rightarrow PQ^2 = 144$

$\Rightarrow PQ = 12cm$

Length of direct common tangent is 12cm

Answer 21.

In right ΔBAC,

$$BC^2 = AC^2 + AB^2$$
$$AC^2 = 13^2 - 5^2$$
$$AC^2 = 169 - 25$$
$$AC^2 = 144$$
$$AC = 12$$

Let $OP = OQ = r$ (say) (radius of same circle)

$\angle OQP = \angle OPQ = 90^\circ$ (radius is $\perp$ to tangent at the point of contact)

$\therefore$ OPAQ is a square.

$AQ = AP = OP = OQ = r$

$BQ = BR = 5 - r$ ---(1) {length of tangents drawn from an external point

$PC = CR = 12 - r$ –(2) to a circle are equal}

$BC = CR + BR$

$13 = 12 - r + 5 - r$ [from (1) and (2)]

$2r = 4$

$r = 2$

Thus, radius of the circle is 2cm.

Answer 22.

$\angle OAP = \angle OBP = 90^\circ$ (radius is $\perp$ to tangent at the point of contact)

In right ΔOAP,

$$OP^2 = OA^2 + AP^2$$
$$OP^2 = 5^2 + 12^2 = 25 + 144 = 169$$
$$OP = 13\text{cm}$$

In right ΔOBP,

$$OP^2 = OB^2 + BP^2$$
$$BP^2 = 13^2 - 3^2$$
$$BP^2 = 169 - 9 = 160$$
$$BP = 4\sqrt{10}\text{cm}$$